Oscillating rope

A heavy uniform rope of length L is suspended vertically from the ceiling. The rope may oscillate around its equilibrium position with different natural frequencies, which will be denoted by f_i $(i = 1, 2, ...)$ in an ascending order. The figure below presents the shape of the rope in the first three natural vibrations, as obtained by a computer simulation. Note that the horizontal and vertical scales in the figures are not equal. You may assume that the actual lateral displacement of the rope is much smaller than its length (small-amplitude approximation).

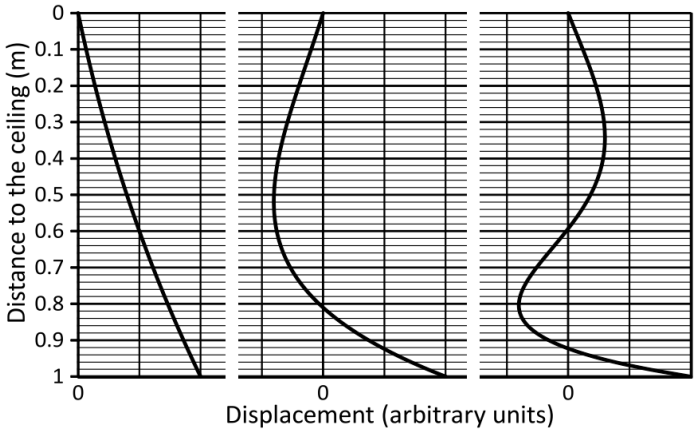

Figura 1: Shapes of the oscillating rope for the first three natural vibrations (i = 1,2,3... from left to right).

a) Develop a simplified model, which will allow you to estimate the frequency f_1 of the first (fundamental) vibration of the rope. Hence, calculate approximately f_1 for a rope of length $L = 1.0\ m$. (Assume that $g = 9.8\ m/s$.)

b) Take necessary readings from the figure to estimate the frequency ratio f_1:f_2:f_3.