Solution - Disk in gas

The initial pressure on the thermal insulating layer is $P_0 = nk_BT_0$, where n is number density of the gas. It originates from multiplying the flux $j_0 \propto v_{x0}$ and momentum that one molecule transfers $p_0 = 2mv_{x0}$ (elastic collision), where v_{x0} is the normal component of molecule's velocity, and taking the average $(\overline{2v_{x0}^2} \propto T_0)$. When applying the same idea to the surface with good thermal contact, we find out that the flux remains the same, although the momentum increases:

$$p_1 = m(v_{x0} + v_{x1}) \approx mv_{x1},$$

where v_{x1} is the normal velocity component of the molecule flying away from the disk. Thus for pressure P_1:

$$\frac{P_1}{P_0} = \frac{\overline{v_{x0}v_{x1}}}{\overline{2v_{x0}^2}} \approx \frac{\sqrt{T_1T_0}}{T_0},$$

which is correct to some numerical coefficient of the order of one.

The net force acting on the disk:

$$F = (P_1 - P_0)S \approx Snk_B\sqrt{T_0T_1},$$

and then the initial acceleration:

$$a_0 \approx \frac{Snk_B}{M}\sqrt{T_0T_1} = \frac{S\rho k_B}{mM}\sqrt{T_0T_1}.$$

Since $P_1 \gg P_0$, the disk will accelerate until its speed becomes of the order of average gas molecules speed. After the velocity v of the disc becomes on the order of $v_0 = \sqrt{k_BT_0/m}$, the flux of molecules reaching the backside $j(v)$ decays faster than exponentially due to the nature of the molecular velocity distribution in the ideal gas (for example, $j(2v_0) \approx 10^{-3}j_0$ and $j(3v_0) \approx 10^{-6}j_0$). That leads to a proportional decrease in a propelling pressure P_1. In order to compensate for an initial bias $\sqrt{T_1/T_0} \approx 30$, it will take around a factor of one on the velocity of the disk. Therefore the maximum velocity of the disk:

$$v_{\max} \approx v_0 = \sqrt{\frac{k_BT_0}{m}}.$$

Here we assumed that the disk will not cool close to T_0 before it reaches the maximum velocity. Let us show it. The acceleration time is approximately:

$$t_a \approx \frac{v_{\max}}{a_0} \approx \frac{M\sqrt{mk_BT_O}}{S\rho k_B\sqrt{T_0T_1}} = \frac{M}{\rho S}\sqrt{\frac{m}{k_BT_1}}$$

Since the power of heat removal P_{th} is maximal at the beginning (at zero velocity), we can upper-bound estimate the time for the disk to cool as $t_c = Q/P_{th}$, where Q is the total heat of the disk. The initial thermal power of heat removal can be estimated as:

$$P_{th} \approx Sj_0 \times k_BT_1 \approx Snk_B\sqrt{T_0T_1}\sqrt{\frac{k_BT_1}{m}}$$

and the total heat $Q \approx N k_B T_1$. Given $M \approx Nm$, we obtain:

$$t_c \approx \frac{(M/m) k_B T_1}{S n k_B T_1 \sqrt{k_B T_0/m}} = \frac{M}{\rho S}\sqrt{\frac{m}{k_B T_0}}$$

Finally, $t_a/t_c \approx \sqrt{T_0/T_1} \ll 1$, and indeed disk will not cool significantly before it reaches the velocity about v_0.

Grading scheme. Indented lines show partial points for partially correct solutions

Initial acceleration (5 pts)

$P_0 = nkT = j_0 \times \Delta p_0$, $\Delta p_0 = 2mv_{x0}$ .... **2 pts**

   $P \propto kT$ .... 1 pts

$\Delta p_1 = mv_{x1}$, $P_1 \approx nk\sqrt{T_0 T_1}$ .... **2 pts**

   $P_1 = nkT_1$, no points for a_0 further .... 1 pts

   only $\Delta p_1 = mv_{x1}$ .... 1 pts

Answer for a_0 .... **1 pts**

   if some slight mistake .... 0.5 pts

Special rule

   if $\langle v_{x0} \rangle = \sqrt{3k_B T/m}$ (student doesn't understand the difference between velocity of the molecule and the component of the velocity) .... −0.5 pts

Maximal velocity (4 pts)

P_1 and P_0 depend on the velocity of the disk, P_1 drops significantly if $v \approx v_0$, thus $v_{\max} \approx v_0$ .... **4 pts**

   $P_0' \approx \rho v^2$, but P_1 stays the same, thus $v_{\max} \approx v_0\sqrt{T_1/T_0}$ .... 2 pts

   Student understands that at least some pressure depends on the velocity of the disk .... 1 pts

   The velocity is maximal when disk cools to T_0 0 pts

Justification of slow cooling (1 pts)

Estimation of times t_a and t_c given..................... **1 pts**

   Student explicitly writes that $T_1' \approx T_1$ but doesn't prove it .... 0.5 pts