

Solution - Staircase

A Since $n = -y/h = (x/\lambda)^{2/3}$, $x(n) = n^{2/3}\lambda$. The distance between the steps is:

$$d_n = x(n+1) - x(n) \approx \frac{dx}{dn} = \frac{2}{3}\lambda n^{-1/3} = n^{-1/3} \cdot 30 \mu\text{m}.$$

B Equilibrium energy value, being minimum, must be stable against small perturbations of the crystal shape. Allowed are perturbations which conserve the total volume of the crystal. In other words a small horizontal displacement of one step must be accompanied by an equal and opposite displacement of another step.

The energy change $\varepsilon_n(\delta)$ associated with a small horizontal displacement δ of the n -th step is:

$$\varepsilon_n(\delta) = \mu \left((d_n + \delta)^\nu - d_n^\nu + (d_{n+1} + \delta)^\nu - d_{n+1}^\nu \right) \approx \mu\nu(d_n^{\nu-1} - d_{n+1}^{\nu-1})\delta.$$

In order for $\varepsilon_n(\delta) + \varepsilon_m(-\delta)$ to be zero for arbitrary n and m it is necessary to require that the factor in the parentheses does not depend on n :

$$d_n^{\nu-1} - d_{n+1}^{\nu-1} = \text{const.}$$

Substituting $d \propto n^{-1/3}$, we get¹:

$$\begin{aligned} n^{(1-\nu)/3} - (n+1)^{(1-\nu)/3} &\approx \frac{1-\nu}{3} n^{\frac{1-\nu}{3}-1} = \text{const}, \\ \frac{1-\nu}{3} - 1 &= 0 \implies \nu = -2. \end{aligned}$$

The interaction energy corresponds to that of two dipoles in 2D:

$$E(d) \propto \frac{1}{d^2}.$$

¹Trivial solutions $\nu = 0$ and $\nu = 1$ imply that the total energy within given constraints does not depend on the shape of the crystal.